### *SUPPLY, DEMAND AND THE EQUILIBRIUM MARKET PRICE*

In a free market economy the supply curve for a commodity is the number of items of a product that can be made available at different prices, and the demand curve is the number of items the consumer will buy at different prices.

As the price of a product increases, its demand decreases and supply increases. On the other hand, as the price decreases the demand increases and supply decreases. The **equilibrium price** is reached when the demand equals the supply.

◆***Example 5*** The supply curve for a product is $y = 35x - 140$ and the demand curve for the same product is $y = -25x + 340$, where x is the price and y the number of items produced. Find the following.

a) How many items will be supplied at a price of \$10?

b) How many items will be demanded at a price of \$10?

c) Determine the equilibrium price.

d) How many items will be produced at the equilibrium price?

***Solution:*** a) We substitute $x = 10$ in the supply equation, $y = 35x - 140$;
the answer is $y = 35(10) - 14\,0 = 210$ items are supplied if the price is \$10.

b) We substitute $x = 10$ in the demand equation, $y = -25x + 340$;
the answer is $y = -25(10) + 340 = 90$ items are demanded if the price is \$10.

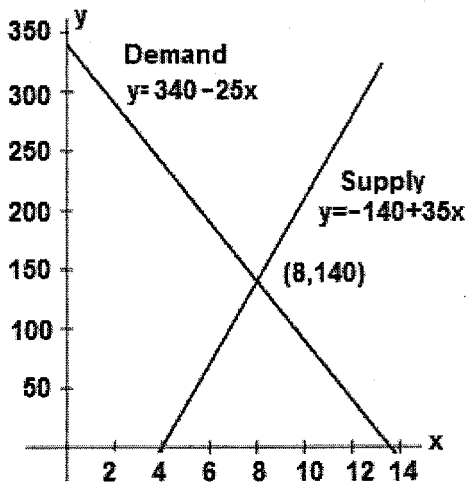
c) By letting the supply equal the demand, we get

$$35x - 140 = -25x + 340$$

$$60x = 480$$

$$x = \$8$$

d) We substitute $x = 8$ in either the supply or the demand equation; we get $y = 140$.




The graph shows the intersection of the supply and the demand functions and their point of intersection, (8, 140).

Interpretation:
At equilibrium, the price is \$8 per item, and 140 items are produced by suppliers and purchased by consumers.

## 5.1 Exponential Growth and Decay Models

*In this section, you will learn to*

1. *recognize and model exponential growth and decay*
2. *compare linear and exponential growth*
3. *distinguish between exponential and power functions*

### *COMPARING EXPONENTIAL AND LINEAR GROWTH*

Consider two social media sites which are expanding the number of users they have:

- Site A has 10,000 users, and expands by adding 1,500 new users each month
- Site B has 10,000 users, and expands by increasing the number of users by 10% each month.

The number of users for Site A can be modeled as linear growth. The number of users increases by a constant number, 1500, each month. If x = the number of months that have passed and y is the number of users, the number of users after x months is y = 10000+1500x.

For site B, the user base expands by a constant percent each month, rather than by a constant number. Growth that occurs at a constant percent each unit of time is called exponential growth.

We can look at growth for each site to understand the difference. The table compares the number of users for each site for 12 months. The table shows the calculations for the first 4 months only, but uses the same calculation process to complete the rest of the 12 months.

| Month | Users at Site A | Users at Site B |
|---|---|---|
| 0 | 10000 | 10000 |
| 1 | 10000 + 1500 = 11500 | 10000 + 10% of 10000<br>= 10000 + 0.10(10000)<br>=10000(1.10) = 11000 |
| 2 | 11500 + 1500 = 13000 | 11000 + 10% of 11000<br>= 11000 + 0.10(11000)<br>=11000(1.10) = 12100 |
| 3 | 13000 + 1500 = 14500 | 12100 + 10% of 12100<br>= 12100 + 0.10(12100)<br>=12100(1.10) = 13310 |
| 4 | 14500 + 1500 = 16000 | 13310 + 10% of 13310<br>= 13310 + 0.10(13310)<br>=13310(1.10) = 14641 |
| 5 | 17500 | 16105 |
| 6 | 19000 | 17716 |
| 7 | 20500 | 19487 |
| 8 | 22000 | 21436 |
| 9 | 23500 | 23579 |
| 10 | 25000 | 25937 |
| 11 | 26500 | 28531 |
| 12 | 28000 | 31384 |

Note: In Example 3, we needed to state the answer to several decimal places of precision to remain accurate. Evaluating the original function using a rounded value of t = 7 years gives a value that is close to 3000, but not exactly 3000.

$$y = 5000(0.93)^7 = 3008.5 \text{ deer}$$

However using t =7.039 years produces a value of 3000 for the population of deer

$$y = 5000(0.93)^{7.039} = 3000.0016 \approx 3000 \text{ deer}$$

◆***Example 4*** A video posted on YouTube initially had 80 views as soon as it was posted. The total number of views to date has been increasing exponentially according to the exponential growth function $y = 80e^{0.12t}$, where t represents time measured in days since the video was posted. How many days does it take until 2500 people have viewed this video?

***Solution***: Let y be the total number of views t days after the video is initially posted. We are given that the exponential growth function is $y = 80e^{0.12t}$ and we want to find the value of t for which y = 2500. Substitute y = 2500 into the equation and use natural log to solve for t.

$$2500 = 80e^{0.12t}$$

Divide both sides by the coefficient, 80, to isolate the exponential expression.

$$\frac{2500}{80} = \frac{80}{80}e^{0.12t}$$

$$31.25 = e^{0.12t}$$

Rewrite the equation in logarithmic form

$$0.12t = \ln(31.25)$$

Divide both sides by 0.12 to isolate t; then use your calculator and its natural log function to evaluate the expression and solve for t.

$$t = \frac{\ln(31.25)}{0.12}$$

$$t = \frac{3.442}{0.12}$$

$$t \approx 28.7 \text{ days}$$

This video will have 2500 total views approximately 28.7 days after it was posted.

***SECTION 7.3 PROBLEM SET: PERMUTATIONS***

Do the following problems using permutations.

| | |
|---|---|
| 1) How many three-letter words can be made using the letters {a, b, c, d, e} if no repetitions are allowed? | 2) A grocery store has five checkout counters, and seven clerks. How many different ways can the 7 clerks be assigned to the 5 counters? |
| 3) A group of fifteen people who are members of an investment club wish to choose a president, and a secretary. How many different ways can this be done? | 4) Compute the following.<br>a) 9P2<br>b) 6P4<br>c) 8P3<br>d) 7P4 |
| 5) In how many ways can the letters of the word CUPERTINO be arranged if each letter is used only once in each arrangement? | 6) How many permutations of the letters of the word PROBLEM end in a vowel? |
| 7) How many permutations of the letters of the word SECURITY end in a consonant? | 8) How many permutations of the letters PRODUCT have consonants in the second and third positions? |