

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Let  $\mathbf{A} = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$ , and  $\mathbf{B} = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$ .  
What value(s) of  $k$ , if any, will make  $\mathbf{A}\mathbf{B} = \mathbf{B}\mathbf{A}$ ?

2. Let  $\mathbf{u} = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix}$ , and  $\mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ . Compute  $\mathbf{u}^T\mathbf{v}$ ,  $\mathbf{v}^T\mathbf{u}$ ,  $\mathbf{u}\mathbf{v}^T$ , and  $\mathbf{v}\mathbf{u}^T$ .

3. Use an augmented matrix and the rref function to find the inverse of the matrix  $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

4. Solve the given matrix equation using an inverse matrix computed in problem 3.

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$$

5. T is a linear transformation from  $\mathbb{R}^2$  into  $\mathbb{R}^2$ .

$$T(x_1, x_2) = (-5x_1 + 9x_2, 4x_1 - 7x_2)$$

a. Find the associated matrix transformation for the following

b. Show that T is invertible and find a formula for  $T^{-1}$  in the form  $T^{-1}(x_1, x_2) =$

6. T is a linear transformation from  $\mathbb{R}^3$  into  $\mathbb{R}^3$ .

$$T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 + x_3, x_1 + 2x_2 + 2x_3)$$

a. Find the associated matrix transformation for the following

b. Show that T is invertible and find a formula for  $T^{-1}$  in the form  $T^{-1}(x_1, x_2, x_3) =$