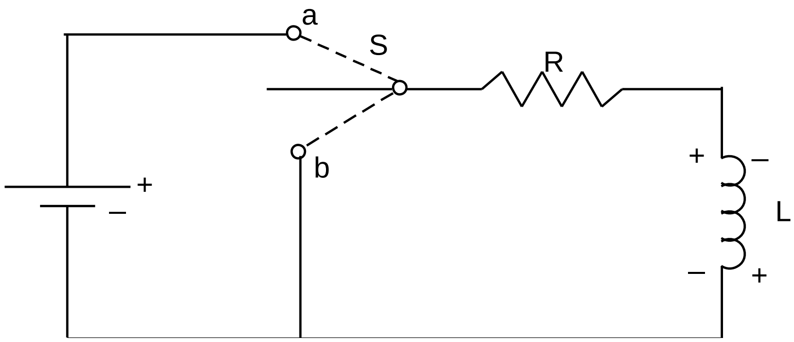
RL Circuits

Consider the RL-Circuit shown below with the switch S initially open.



$$\varepsilon_L = -L\frac{dI}{dt}$$

Switch in Position 'a'

$$\sum V_{loop} = 0$$

$$V - IR - L\frac{dI}{dt} = 0$$

$$\frac{dI}{dt} = \frac{V}{L} - \frac{IR}{L}$$

At t=0 when S is closed $I = 0$:

$\boxed{\frac{dI}{dt} = \frac{V}{L}}$ At t = 0

The larger the inductance *L*, the smaller dI/dt and the larger the opposition to the increase in current and thus the more slowly the current increases.

As I increase dI/dt → 0 and the current reaches its steady state value:

$$0 = \frac{V}{L} - \frac{IR}{L}$$

$\boxed{I = \frac{V}{R}}$ Steady-state current

$$\frac{dI}{dt} = \frac{VR}{LR} - \frac{IR}{L} = -\frac{R}{L}\left(I - \frac{V}{R} \right)$$

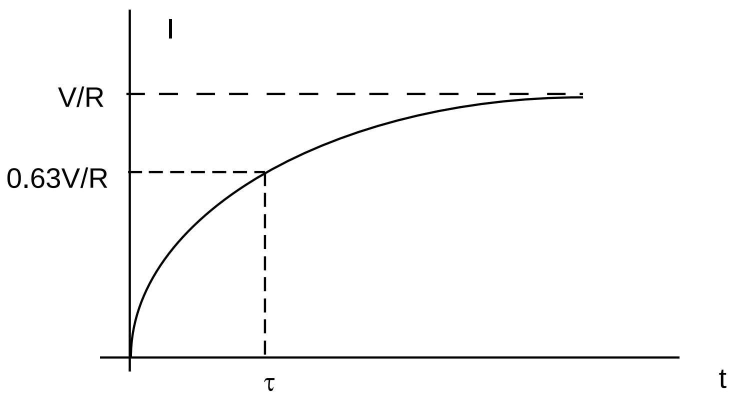
$$\int_0^I \frac{dI}{I - \frac{V}{R}} = -\frac{R}{L}\int_0^t dt$$

$$\ln\left(I - \frac{V}{R} \right)_0^I = -\frac{R}{L}t$$

$$\ln\left(I - \frac{V}{R} \right) - \ln\left(-\frac{V}{R} \right) = -\frac{t}{\tau}$$

where $\tau = \frac{L}{R}$ (time constant)

$$\boxed{I(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}} \right)}$$



$$\frac{dI}{dt} = -\frac{V}{R}\left(-\frac{1}{\tau}\right)e^{-\frac{t}{\tau}}$$

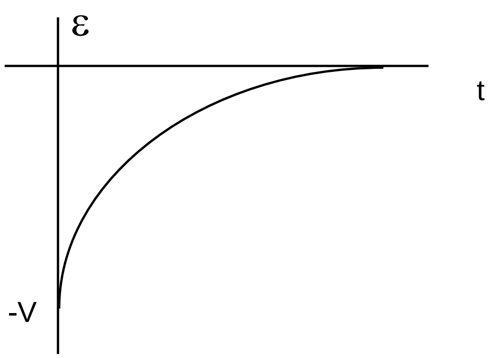
$$\boxed{\frac{dI}{dt} = \frac{V}{L}e^{-\frac{t}{\tau}}}$$

$$\varepsilon_L = -L\frac{dI}{dt}$$

$$\varepsilon_L = -L\left(\frac{V}{L}\right)e^{-\frac{t}{\tau}}$$

$$\varepsilon_L = -Ve^{-\frac{t}{\tau}}$$

$$\varepsilon_L(0) = -V$$



Switch in Position 'b'

$$\sum V_{loop} = 0$$

$$-IR - L\frac{dI}{dt} = 0$$

$$\frac{dI}{I} = -\frac{R}{L}dt$$

$$\int_{V/R}^{I} \frac{dI}{I} = -\frac{R}{L}\int_{0}^{t} dt$$

$$\boxed{I(t) = \frac{V}{R}e^{-\frac{t}{\tau}}}$$