

Rule Name	What You Need	How to Cite (Annotate) It	Assumption Set	Example
&I	2 lines, $m$ and $k$	$m, k \&I$	Include all assumptions from lines $m$ and $k$	1 (1) A A 2 (2) B A 1,2 (3) A&B 1,2&I
&E	1 line $m$ : must be a conjunction	$m \&E$	Include all assumptions from line $m$	1 (1) A&B A 1 (2) A 1&E
$\vee I$	1 line $m$ : can be anything	$m \vee I$	Include all assumptions from line $m$	1 (1) P A 1 (2) P $\vee$ Q 1 $\vee I$
$\vee E$	2 lines, $m$ and $k$ : $m$ must be a disjunction, and $k$ must be the denial (negation) of one of the disjuncts in $m$ . Conclude remaining disjunct.	$m, k \vee E$	Include all assumptions from lines $m$ and $k$	1 (1) P $\vee$ Q A 2 (2) $\sim$ Q A 1,2 (3) P 1,2 $\vee E$
$\rightarrow I$	2 lines, $m$ and $k$ : $m$ must be an assumption, and $k$ can be any line. Conclude a conditional of the form ( $m \rightarrow k$ ), and discharge the assumption at $m$ .	$k \rightarrow I (m)$	Include all assumptions at $k$ except for $m$	1 (1) P A . . . 2,3 (5) Q&R 2,3 &I 2,3 (6) P $\rightarrow$ (Q&R) 5 $\rightarrow I$ (1)
$\rightarrow E$	2 lines, $m$ and $k$ : $m$ must be a conditional, and $k$ must be the antecedent of the conditional at $m$ . Conclude consequent of conditional at $m$ .	$m, k \rightarrow E$	Include all assumptions from lines $m$ and $k$	1 (1) P $\rightarrow$ Q A 2 (2) P A 1,2 (3) Q 1,2 $\rightarrow E$
$\leftrightarrow I$	2 lines, $m$ and $k$ : $m$ must be a conditional of the form (P $\rightarrow$ Q), and $k$ must be a conditional of the form (Q $\rightarrow$ P)	$m, k \leftrightarrow I$	Include all assumptions from lines $m$ and $k$	1 (1) P $\rightarrow$ Q A 2 (2) Q $\rightarrow$ P A 1,2 (3) P $\leftrightarrow$ Q 1,2 $\leftrightarrow I$
$\leftrightarrow E$	1 line $m$ : must be a biconditional of the form (P $\leftrightarrow$ Q). Conclude either P $\rightarrow$ Q or Q $\rightarrow$ P.	$m \leftrightarrow E$	Include all assumptions from line $m$	1 (1) P $\leftrightarrow$ Q A 1 (2) P $\rightarrow$ Q 1 $\leftrightarrow E$
RAA	2 lines, $m$ and $k$ , that contradict each other, and a line $l$ that is an assumption ( $l$ can be the same as lines $m$ or $k$ ). Conclude the denial of $l$ , and discharge the assumption at $l$ .	$m, k \text{ RAA } (l)$	Include all assumptions at $m$ and $k$ except for $l$ .	1 (1) P A 2 (2) $\sim$ P A 1 (3) $\sim\sim$ P 1,2 RAA (2)
$\forall I$	1 line $m$ : must be a sentence with at least one name embedded in a predicate letter. This name cannot appear in any of the lines in the assumption set at $m$ . Conclude a universally quantified sentence with the name replaced by an universally bound variable.	$m \forall I$	Include all assumptions from line $m$	1 (1) $\forall x$ Px A 1 (2) Pa 1 $\forall E$ 1 (3) Pa v Ga 2 $\vee I$ 1 (4) $\forall x$ (PxvGx) 3 $\forall I$
$\forall E$	1 line $m$ : must be a universally quantified sentence. Conclude any instance of $m$ .	$m \forall E$	Include all assumptions from line $m$	1 (1) $\forall x$ Px A 1 (2) Pa 1 $\forall E$
$\exists I$	1 line $m$ : must be a sentence with at least one name embedded in a predicate letter. Conclude an existentially quantified sentence with the name replaced by an existentially bound variable.	$m \exists I$	Include all assumptions from line $m$	1 (1) Pa A 1 (2) $\exists x$ Px 1 $\exists I$
$\exists E$	3 lines $m, k, l$ : $m$ must be an existentially quantified sentence, $k$ must be an assumed instance of $m$ , and $l$ must be a sentence that does not contain the name instantiated at $k$ . This name cannot appear in sentence $m$ , sentence $k$ , or in any of the assumptions at $k$ except for $m$ . Conclude a sentence that repeats the content at line $l$ , and discharge $m$ .	$m, l \exists E (k)$	Include all assumptions from lines $m$ and $l$ except for $k$ .	1 (1) $\exists x$ Px A 2 (2) Pa A 2 (3) PavGa 2 $\vee I$ 2 (4) $\exists x$ (Px v Gx) 3 $\exists I$ 1 (5) $\exists x$ (Px v Gx) 1,4 $\exists E$ (2)