THE SPEED OF LIGHT

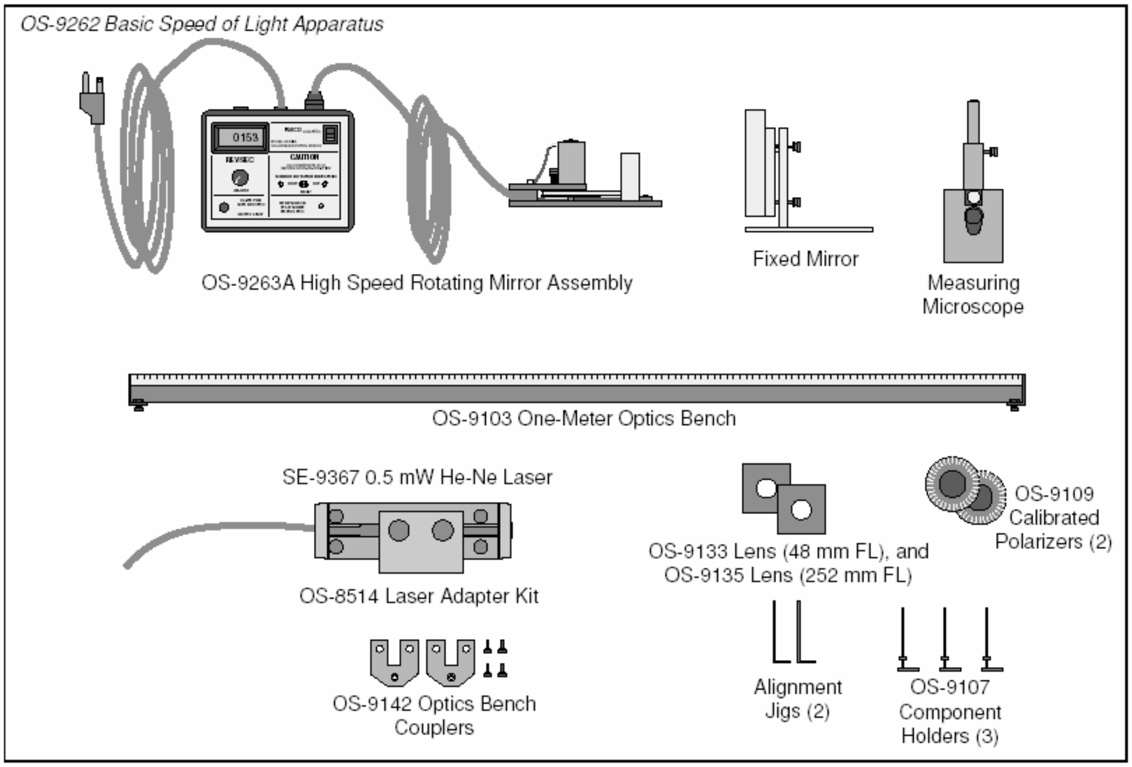
Introduction

According to Einstein's Principle of Relativity the speed of light in a vacuum has the same value, $c = 3.00 \times 10^8$ m/s, in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light. Prior to Einstein's special theory of relativity there had been a great deal of time and effort invested in measuring changes in the speed of light to confirm the 'ether theory'. Although the Michelson-Morley Experiment proved that there was no 'ether', the result had far reaching consequences; it indirectly confirmed Einstein's Principle of Relativity.

In this experiment light is timed as it travels over a known distance. A light beam is initially reflected from a rotating mirror to a fixed mirror, where it reflects back to the rotating mirror. When the light beam returns to the rotating mirror, this mirror will be in a slightly different orientation. As a result the returning beam will reflect from this mirror along a slightly different line that the initial beam of light had. Measurement of the angle between these two beams of light is used to time the trip back and forth and thus obtain a numerical value for the speed of light.

Equipment (*see following page*)

- high speed rotating mirror
- fixed mirror
- measuring microscope
- 0.5 mW He-Ne laser
- one-meter optics bench
- laser alignment bench
- tape measure
- optics bench couplers
- 1 lens (48 mm FL)
- 1 lens (252 mm FL)
- 2 calibrated polarizers
- 3 component holders
- 2 alignment jigs


OS-9262 Basic Speed of Light Apparatus

Theory

The light beam from a laser is focused by lens L_1 to form a point image at h. The beam then passes through a beam splitter, another lens L_2, and travels to a rotating mirror M_R. After reflecting from the rotating mirror the beam travels to a fixed mirror M_F. Lens L_2 is adjusted so that a point image is formed at the fixed mirror M_F. After reflecting from the fixed mirror, the beams returns to the rotating mirror. It then reflects from the rotating mirror, passes through lens L_2, and reflects from the beam splitter to form a final point image at h'. This final image is observed with a microscope. See figure 1.

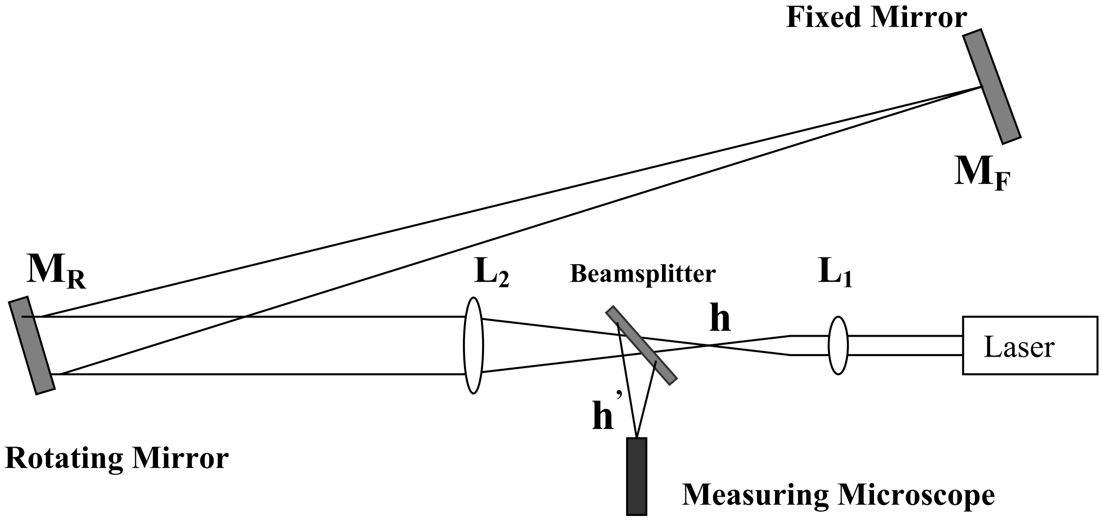


Figure 1

Figure 2 below shows the path of the beam of light, from the laser to M_F, when M_R is at an angle θ to the vertical. In this case, the angle of incidence of the light path as it strikes M_R is also θ and, since the angle of incidence equals the angle of reflection, the angle between the incident and reflected ray is just 2θ. As shown in the figure, the beam of light strikes M_R at the point labeled S.

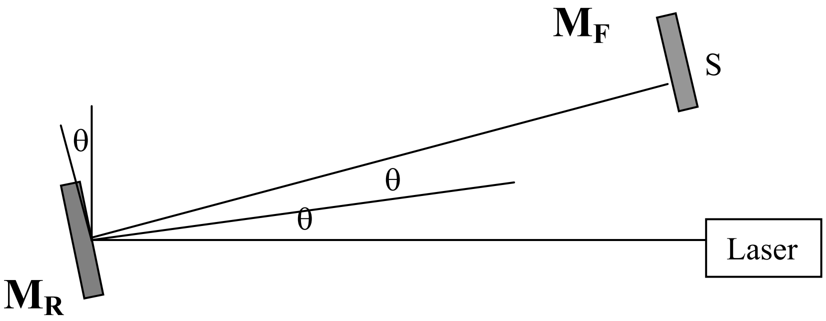


Figure 2

Figure 3 shows the path of the beam of light if it leaves the laser at a slightly later time, when M_R is at an angle $\theta_1 = \theta + \Delta\theta$. The angle of incidence is now equal to $\theta_1 = \theta + \Delta\theta$, so that the angle between the incident and reflected rays is $2\theta_1 = 2(\theta + \Delta\theta)$. This time we label the point where the pulse strikes M_F as S_1.

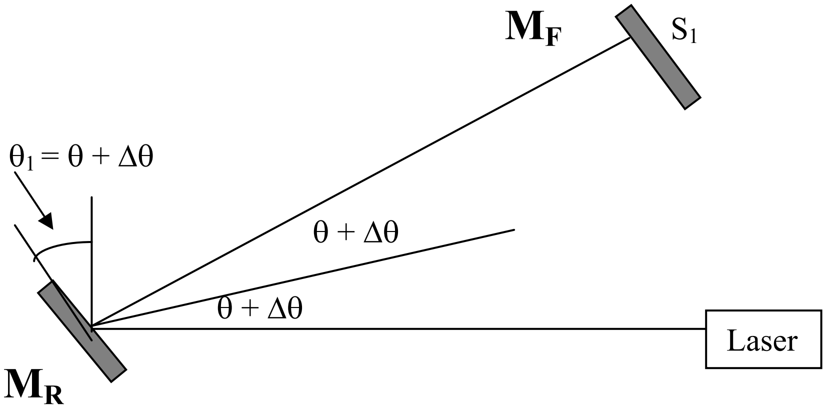


Figure 3

If the distance between M_F and M_R is D, then the distance between S and S_1 can be calculated:

$$\Delta S = S_1 - S = D(2\theta_1 - 2\theta)$$

$$\Delta S = D[2(\theta + \Delta\theta) - 2\theta] = 2D\Delta\theta \qquad (1)$$

From Figure 4 below we see that the image displacement Δh is the object for lens L_2 while ΔS at the fixed mirror M_F is the image for lens L_2.

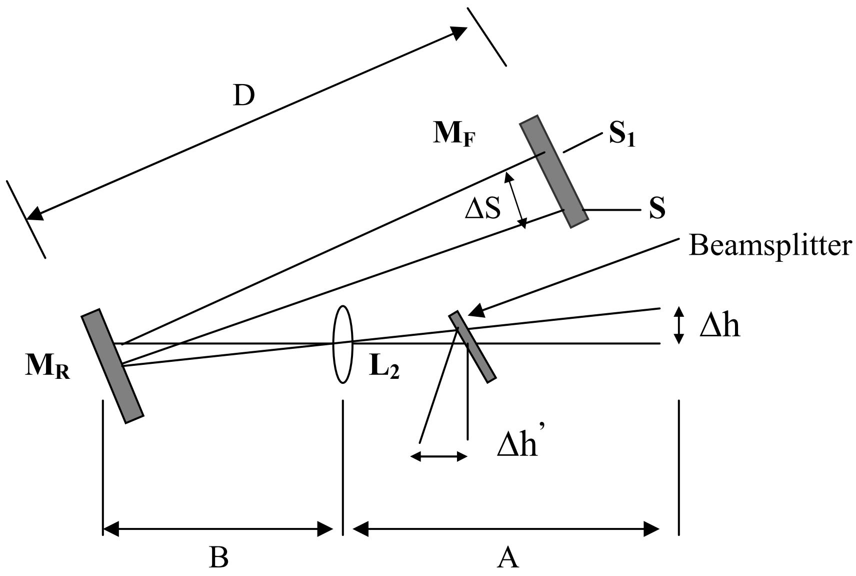


Figure 4

From the thin lens equation we have:

$$\Delta h = \Delta h' = (-\mathbf{i}/\mathbf{o})\Delta S \qquad (2)$$

Here **i** and **o** are the distances of the lens from the image and object, respectively, and the minus sign corresponds to the inversion of the image. Note, that since the distance from the beam splitter to s and s' of figure are the same, then the beam splitter forms a similar image of the same height (Δh = Δh'). Since we are not concerned if the image is inverted or not, then the image displacement (Δh') is given by:

$$\Delta h' = \frac{A}{D+B}\Delta S$$

$$(3)$$

$$\Delta h' = \frac{2DA\Delta\theta}{D+B}$$

The angle $\Delta\theta$ depends on the rotational speed of M_R and on the time it takes the light pulse to travel back and forth between M_R and M_F. The relationship is:

$$\Delta\theta = \frac{2D\omega}{c} \qquad (4)$$

Substituting Eq. (4) into Eq. (3) gives:

$$\Delta h' = \frac{4D^2 A\omega}{c(D+B)} \qquad (5)$$

Eq. (5) can now be rearranged to provide our final equation for the speed of light:

$$c = \frac{4D^2 A\omega}{(D+B)\Delta h'} \qquad (6)$$

Procedure

1. Setup the apparatus and align the system as described in the handout ‘Setup and Alignment’.

2. Before turning on the motor, be sure the lock-screw for the rotating mirror is completely loosened, so the mirror rotates freely by hand. Never run the motor with MAX REV/SEC button pushed for more than one minute at a time, and always allow about one minute between runs for the motor to cool off.

3. With the apparatus aligned and the beam image in sharp focus, set the direction switch in the rotating mirror power supply to CW, and turn on the motor. If the image is not in sharp focus, or if it disappears, adjust the microscope. You should also turn L_2 slightly askew (about 1 - 2°) to improve the image. To get the best image you may need to adjust the microscope and L_2 several times.

4. Let the motor warm up at about 600 rev/s for at least 3 minutes.

5. Slowly increase the speed of rotation and notice how the image deflection increases.

6. Use the ADJUST knob to bring the rotational speed up to about 1000 rev/s. Then push the MAX REV/SEC button and hold it down. When the rotation speed stabilizes, rotate the micrometer knob on the microscope to align the center of the beam image with the cross hair in the microscope that is perpendicular to the direction of deflection.

7. Record the rotational speed, turn off the motor, and record the micrometer reading.

8. Reverse the direction of the mirror rotation by switching the direction switch on the power supply to CCW. Allow the mirror to come to a complete stop before reversing the direction. Then repeat your measurement as in step 3.

9. When adjusted to fit the parameters just measured, the equation for the speed of light becomes:

$$c = \frac{8\pi A D^2 (rev/\sec_{cw} + rev/\sec_{ccw})}{(D+B)(h'_{cw} - h'_{ccw})} \qquad (7)$$

Note: To measure A, measure the distance between L_1 *and* L_2*, then subtract the focal length of* L_1*, 48 mm.*

Analysis

1. Show a step-by-step derivation of equation (7).
2. Take 2 measurement for the speed of light using the procedure described above.
3. Calculate the % error for each measurement.
4. Comment on any systematic or random errors involved in the experiment.

References:

1. PASCO scientific, Instruction Manual and Experiment Guide, 1989

SEE SETUP AND ALIGNMENT FOR THE APPARATUS ON THE FOLLOWING PAGE

Setup and Alignment

The following alignment procedure is tailored for those using the OS-9261A Complete Speed of Light Apparatus. For those using only some of the components in the complete system, the general procedure is the same, though the details depend on the optical components used.

> ➤ **IMPORTANT:** Proper alignment is critical, not only for getting good results, but for getting any results at all. Please follow this alignment procedure carefully. *Allow yourself about three hours to do it properly the first time.* Once you have set up the equipment a few times, you may find that the alignment summary at the end of this section is a helpful guide.

For reference as you set up the equipment, Figure 5 shows the approximate positioning of the components with respect to the metric scale on the side of the Optics Bench. The exact placement of each component depends on the position of the Fixed Mirror (M_F) and must be determined by following the steps of the alignment procedure described below.

All component holders, the Measuring Microscope, and the Rotating Mirror Assembly should be mounted flush against the "fence" of the Optics Bench (Figure 6). This will insure that all components are mounted at right angles to the beam axis.

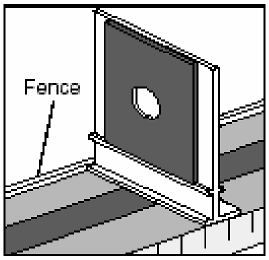


Figure 6: Placing Components Flush Against the Fence for Proper Alignment

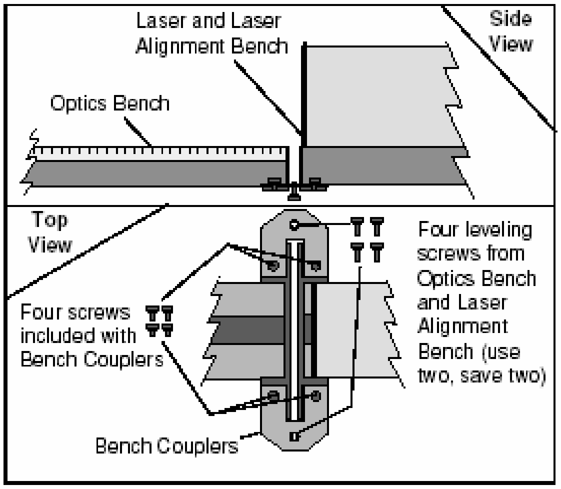
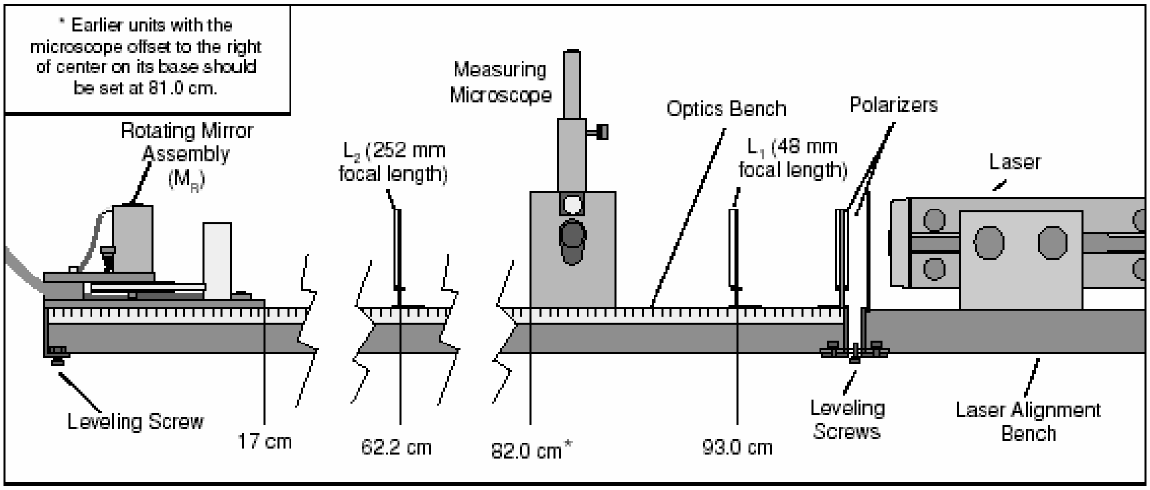


Figure 7: Coupling the Optics Bench and the Laser Alignment Bench



* Earlier units with the microscope offset to the right of center on its base should be set at 81.0 cm.

Figure 5: Equipment Alignment

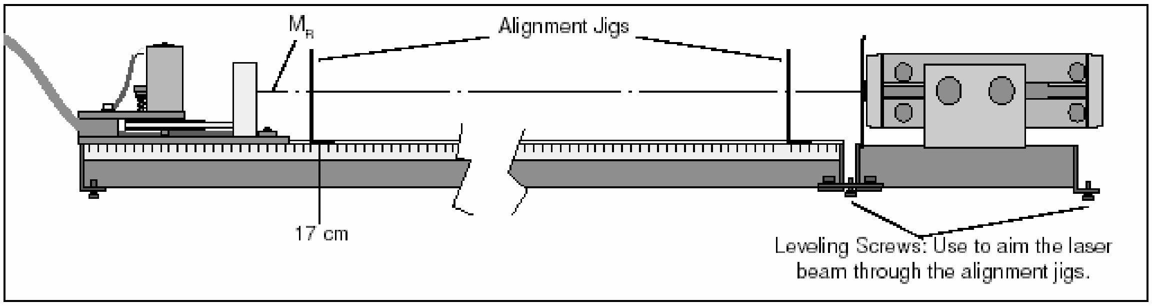


Figure 8: Using the Alignment Jigs to Align the Laser

To Set up and Align the Equipment:

1. Place the Optics Bench on a flat, level surface.
2. Place the Laser, mounted on the Laser Alignment Bench, end-to-end with the Optics Bench, at the end corresponding to the 1-meter mark of the metric scale.
3. Use the Bench Couplers and the provided screws to connect the Optics Bench and the Laser Alignment Bench. Details are shown in Figure 7. Do not yet tighten the screws holding the Bench Couplers. ➤ Note that the leveling screws must be removed from the Optics Bench and from the Laser Alignment Bench to attach the Bench Couplers. Two of the removed leveling screws are then inserted into the threaded holes in the Bench Couplers and are used for leveling.
4. Mount the Rotating Mirror Assembly on the opposite end of the bench. Be sure the base of the assembly is flush against the fence of the Optics Bench and align the front edge of the base with the 17 cm mark on the metric scale of the Optics Bench (see Figure 8).
5. The laser must be aligned so the beam strikes the center of the Rotating Mirror (M_R). Two alignment jigs are provided for this purpose. Place one jig at each end of the Optics Bench as shown in Figure 8, with the edges flush against the fence of the bench. When properly placed, the holes in the jigs define a straight line that is parallel to the axis of the Optics Bench.
6. Turn on the Laser.

> ➤ **CAUTION:** Do not look into the laser beam, either directly or as it reflects from either mirror. Also, when arranging the equipment, be sure the beam path does not traverse an area where someone might inadvertently look into the beam.

7. Adjust the position of the front of the laser so the beam passes directly through the hole in the first jig. (Use the two front leveling screws to adjust the height. Adjust the position of the laser on the Laser Alignment Bench to adjust the lateral position.) Then adjust the height and position of the rear of the laser so the beam passes directly through the hole in the second jig.

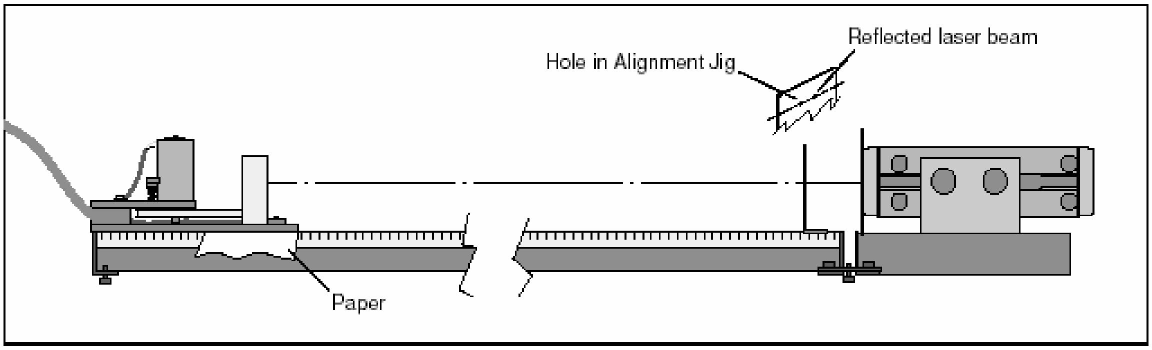


Figure 9: Aligning the Rotating Mirror (M_R)

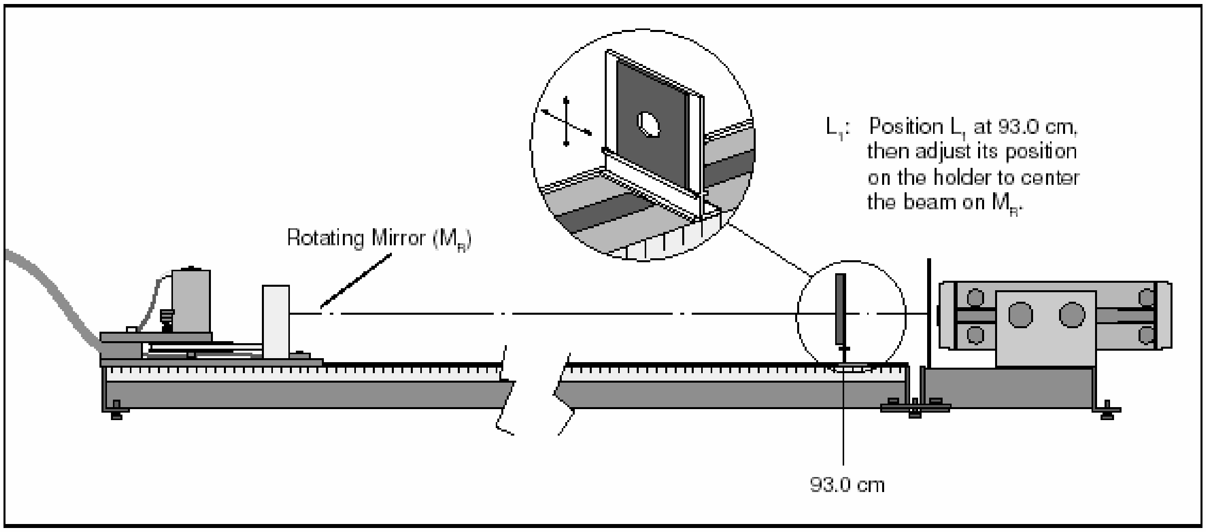


Figure 10: Positioning and Aligning L_1

8. To fix the laser in position with respect to the Optics Bench, tighten the screws on the Bench Couplers. Then recheck the alignment of the laser.

9. Align the Rotating Mirror. $\mathbf{M_R}$ must be aligned so that its axis of rotation is vertical and also perpendicular to the laser beam. To accomplish this, remove the second alignment jig and then rotate $\mathbf{M_R}$ so that the laser beam reflects back toward the hole in the first alignment jig (Figure 9). Be sure to use the reflective side of the mirror. It helps to tighten the lock-screw on the rotating mirror assembly just enough so $\mathbf{M_R}$ holds its position as you adjust its rotation.

   If needed, use pieces of paper to shim between the Rotating Mirror Assembly and the Optics Bench so that the laser beam is reflected back through the hole in the first jig.

10. Remove the first alignment jig.

11. Mount the 48 mm focal length lens ($\mathbf{L_1}$) on the Optics Bench so that the center line of the Component Holder is aligned with the 93.0 cm mark on the metric scale of the bench. Without moving the Component Holder, slide $\mathbf{L_1}$ as needed on the holder to center the beam on $\mathbf{M_R}$ (see Figure 10). Notice that L_1 has spread the beam at the position of $\mathbf{M_R}$.

12. Mount the 252 mm focal length lens ($\mathbf{L_2}$) on the Optics Bench so the center line of the Component Holder aligns with the 62.2 cm mark on the metric scale of the bench. As for $\mathbf{L_1}$ in step 11, adjust the position of $\mathbf{L_2}$ on the Component Holder so that the beam is again centered on $\mathbf{M_R}$.

13. Place the Measuring Microscope on the Optics Bench so that the left edge of the mounting stage is aligned with the 82.0 cm mark on the bench (see Figure 5). The lever that adjusts the tilt of the beam splitter should be on the same side as the metric scale of the Optics Bench. Position this lever so it points directly down.

   ➧ **CAUTION:** Do not look through the microscope until the polarizers have been placed between the laser and the beam splitter in step 19.

   The beamsplitter will slightly alter the position of the laser beam. Readjust $\mathbf{L_2}$ on the Component Holder so the beam is again centered on $\mathbf{M_R}$.

14. Place the Fixed Mirror (M_F) from 2 to 15 meters from M_R, as shown in Figure 11. The angle between the axis of the Optics Bench and a line from $\mathbf{M_R}$ to M_F should be approximately 12 degrees. (If it is greater than 20-degrees, the reflected beam will be blocked by the Rotating Mirror enclosure.) Also be sure that M_F is not on the same side of the optical bench as the micrometer knob, so you will be able to make the measurements without blocking the beam.

➧ **NOTE:** Best results are obtained when M_F is 10 to 15 meters from $\mathbf{M_R}$. See *Notes on Accuracy* near the end of the manual.

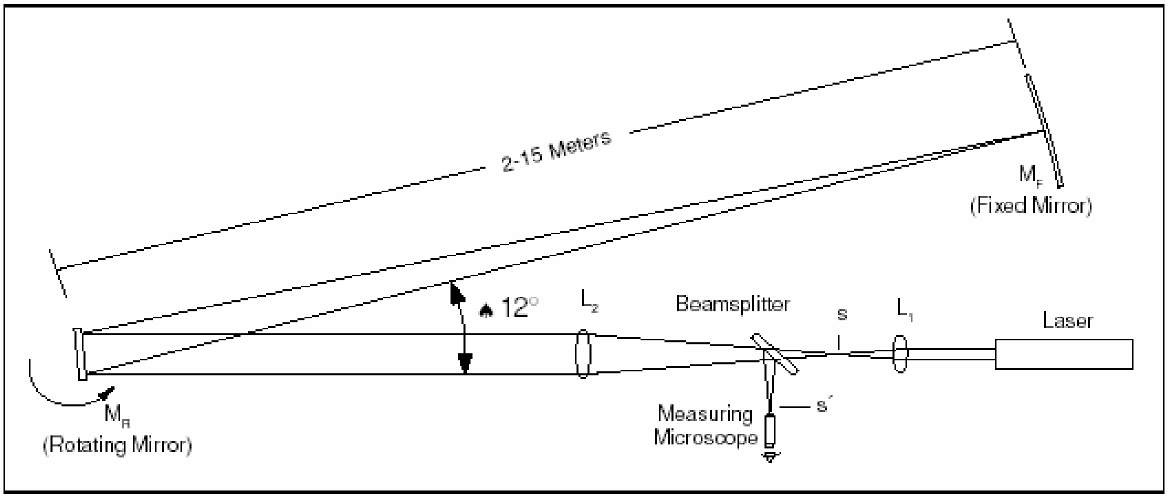


Figure 11: Positioning the Fixed Mirror (M_F)

15. Position M_R so the laser beam is reflected toward M_F. Place a piece of paper in the beam path and "walk" the beam toward M_F, adjusting the rotation of M_R as needed.
16. Adjust the position of M_F so the beam strikes it approximately in the center. Again, a piece of paper in the beam path will make the beam easier to see.
17. With a piece of paper still against the surface of M_F, slide L_2 back and forth along the Optics Bench to focus the beam to the smallest possible point on M_F.
18. Adjust the two alignment screws on the back of M_F so the beam is reflected directly back to the center of M_R. This step is best performed with two people: one adjusting M_F, and one watching the beam position at M_R.
19. Place the polarizers (attached to either side of a single Component Holder) between the laser and L_1. Begin with the polarizers at right angles to each other, than rotate one until the image in the microscope is bright enough to view comfortably.

If you can't find the point image there are several things you can try:

- Vary the tilt of the beamsplitter slightly (no more than a few degrees) and turn the micrometer knob to vary the transverse position of the microscope until the image comes into view.
- Loosen the lock-screw on the microscope. As shown in Figure 13, remove the microscope and place a piece of tissue paper over the tube to locate the beam. Adjust the beamsplitter angle and the micrometer knob to center the point image in the tube of the microscope.
- Slide the Measuring Microscope a centimeter or so in either direction along the axis of the Optics Bench. Be sure that the Microscope stays flush against the fence of the Optics Bench. If this doesn't work, recheck the alignment, beginning with step 1.

20. Bring the cross-hairs of the microscope into focus by sliding the microscope eyepiece up and down.
21. Focus the microscope by loosening the lock-screw and sliding the scope up and down. If the apparatus is properly aligned, you will see the point image through the microscope. Focus until the image is as sharp as possible.

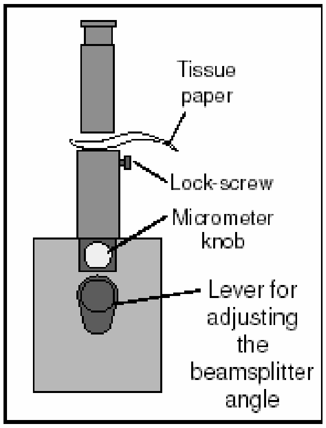


Figure 13: Looking for the Beam Image

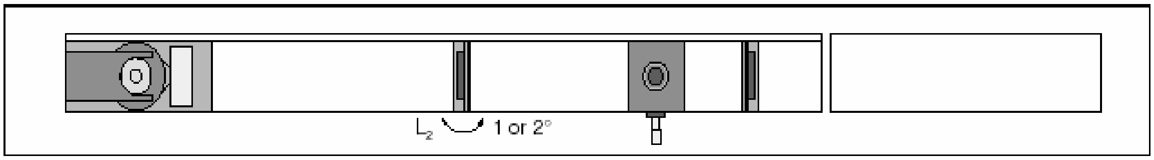


Figure 12: Turning L_2 Slightly Askew to Clean Up the Image

> ➤ **IMPORTANT:** In addition to the point image, you may also see some extraneous beam images resulting, for example, from reflection of the laser beam from L_1. To be sure you are observing the right image point, place a piece of paper between M_R and M_F while you watch the image in the microscope. If the point does not disappear, it is not the correct image.

Cleaning Up the Image

22. In addition to the point image, you may also see interference fringes through the microscope (as well as the extraneous beam images mentioned above). These fringes cause no difficulty as long as the point image is clearly visible. However, the fringes and extraneous beam images can sometimes be removed without losing the point image. This is accomplished by turning L2 slightly askew, so it is no longer quite at a right angle to the beam axis (see Figure 12).

Alignment Summary

(see Figure 14 for approximate component placement)

This summary is for those who are familiar with the equipment and the experiment, and just need a quick reminder of the steps in the alignment procedure. If you have not successfully aligned the apparatus before, we recommend that you take the time to go through the detailed alignment procedure in the preceding section.

1. Align the laser so the laser beam strikes the center of M_R (use the alignment jigs).
2. Adjust the rotational axis of M_R so it is perpendicular to the beam (i.e. as M_R rotates, there must be a position at which it reflects the laser beam directly back into the laser aperture).
3. Insert L_1 to focus the laser beam to a point. Adjust L_1 so the beam is still centered on M_R.
4. Insert L_2 and adjust it so the beam is still centered on M_R.
5. Place the Measuring Microscope in position and, again, be sure that the beam is still centered on M_R.

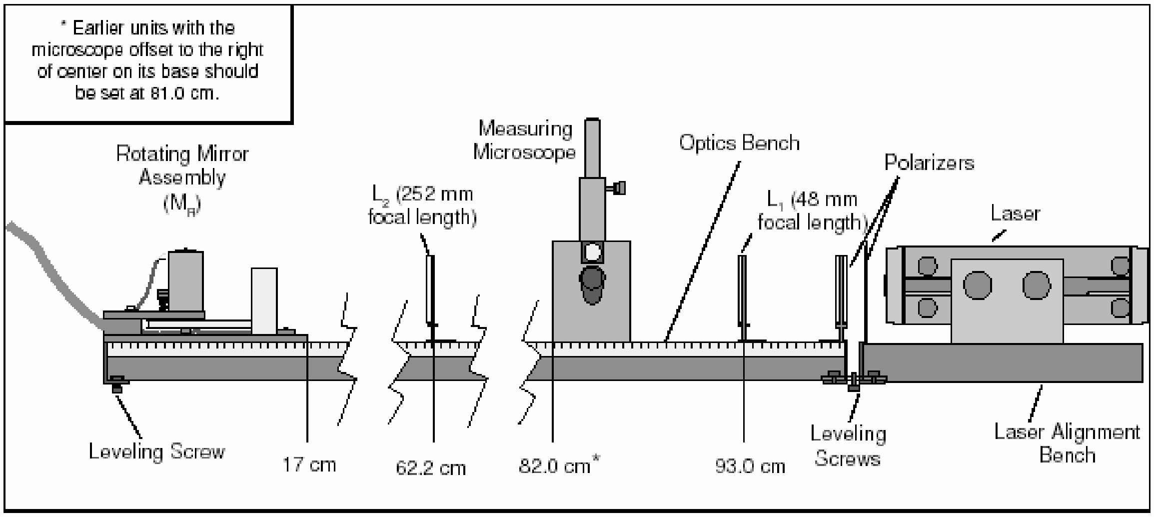

* Earlier units with the microscope offset to the right of center on its base should be set at 81.0 cm.

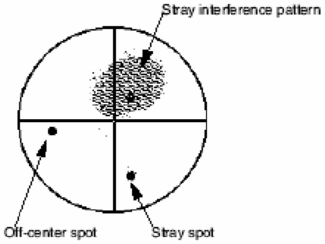
Figure 14: Equipment Alignment

➧ **CAUTION:** Do not look through the microscope until the polarizers have been placed between the laser and the beamsplitter.

6. Position $\mathbf{M_F}$ at the chosen distance from $\mathbf{M_R}$ (2 - 15 meters), so the reflected image from $\mathbf{M_R}$ strikes the center of $\mathbf{M_F}$.
7. Adjust the position of $\mathbf{L_2}$ to focus the beam to a point on $\mathbf{M_F}$.
8. Adjust $\mathbf{M_F}$ so the beam is reflected directly back onto $\mathbf{M_R}$.
9. Insert the polarizers between the laser and the beam splitter.
10. Focus the microscope on the image point.
11. Remove polarizers.

Alignment Hints

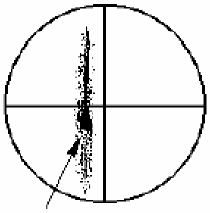
Once you have the microscope focused, it may still be difficult to obtain a good spot. There may be several other lights visible in the microscope besides the spot reflected from the fixed mirror.



The most common of these are stray interference patterns. These are caused by multiple reflections from the surfaces of the lenses, and may be ignored. If necessary, you may be able to eliminate them by angling the lenses 1 – 2°.

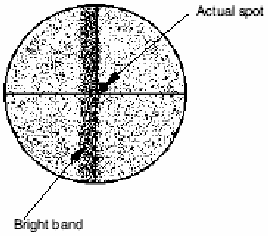
Stray Spots are most often caused by reflections off the window of the rotating mirror housing. To determine which spot is the one you must measure, block the beam path between the rotating mirror and the fixed mirror. The relevant spot will disappear.

If the spot you need to measure is significantly off-center, you can move it by adjusting the angle of the beamsplitter.



Enlongated spot with fringes

Another common problem is a spot that is "stretched" with no easily discernible maxima. Check first to make sure that this is the spot you need by blocking the beam path between the moving and fixed mirrors. If it is, then twist L_2 slightly until the image coalesces into a single spot.



Once the mirror begins to rotate, it is safe to look into the microscope without the polarizers. You will notice that your carefully aligned pattern has changed: now the entire field is covered with a random interference pattern, and there is a bright band down the center of the field. Ignore the interference pattern; there's nothing you can do about it anyway. The band is the image of the laser when, once each rotation, the mirror reflects it into the microscope beamsplitter. This is also unavoidable.

Your actual spot will probably be just to one side of the bright band. You can check for it by blocking and unblocking the beam path between the rotating mirror and fixed mirror and watching to see what disappears.

If you aligned everything ***perfectly***, the spot will be hidden by the bright band; in this case, make sure that you have a spot when the rotating mirror is fixed and is reflecting the laser to the fixed mirror. If you do have the correct spot under stationary conditions, then misalign the fixed mirror *very slightly* (0.004° or less) around the horizontal axis. This will bring the actual spot out from under the bright band.